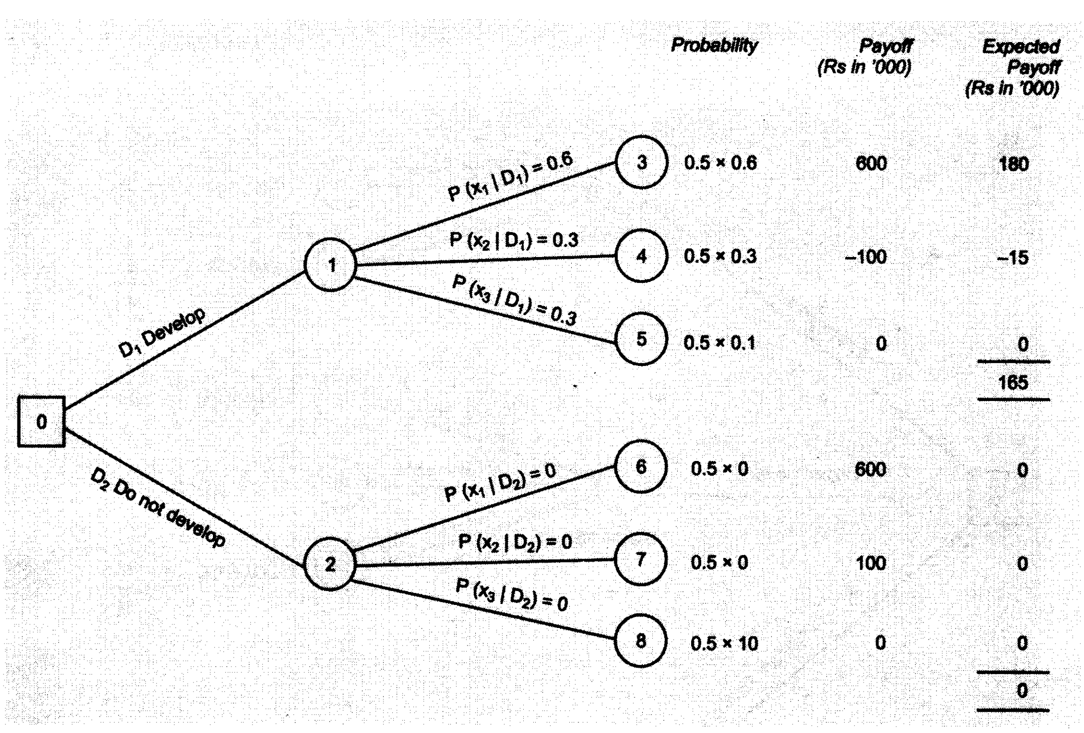


Figure 19.2 Decision Tree

Example 19.17: A glass factory specializing in crystal is developing a substantial backlog and the firm's management is considering three courses of action: Arrange for subcontracting (S_1), begin overtime production (S_2), and construct new facilities (S_3). The correct choice depends largely upon future demand which may be low, medium, or high. By consensus, management ranks the respective probabilities as 0.10, 0.50, and 0.40. A cost analysis reveals the effect upon the profits that is shown in the table below:

Demand	*Probability*	*Course of Action*		
		S_1 (*Subcontracting*)	S_2 (*Begin Overtime*)	S_3 (*Construct Facilities*)
Low (L)	0.10	10	– 20	– 150
Medium (M)	0.50	50	60	20
High (H)	0.40	50	100	200

Show this decision situation in the form of a decision tree and indicate the most preferred decision and the corresponding expected value.

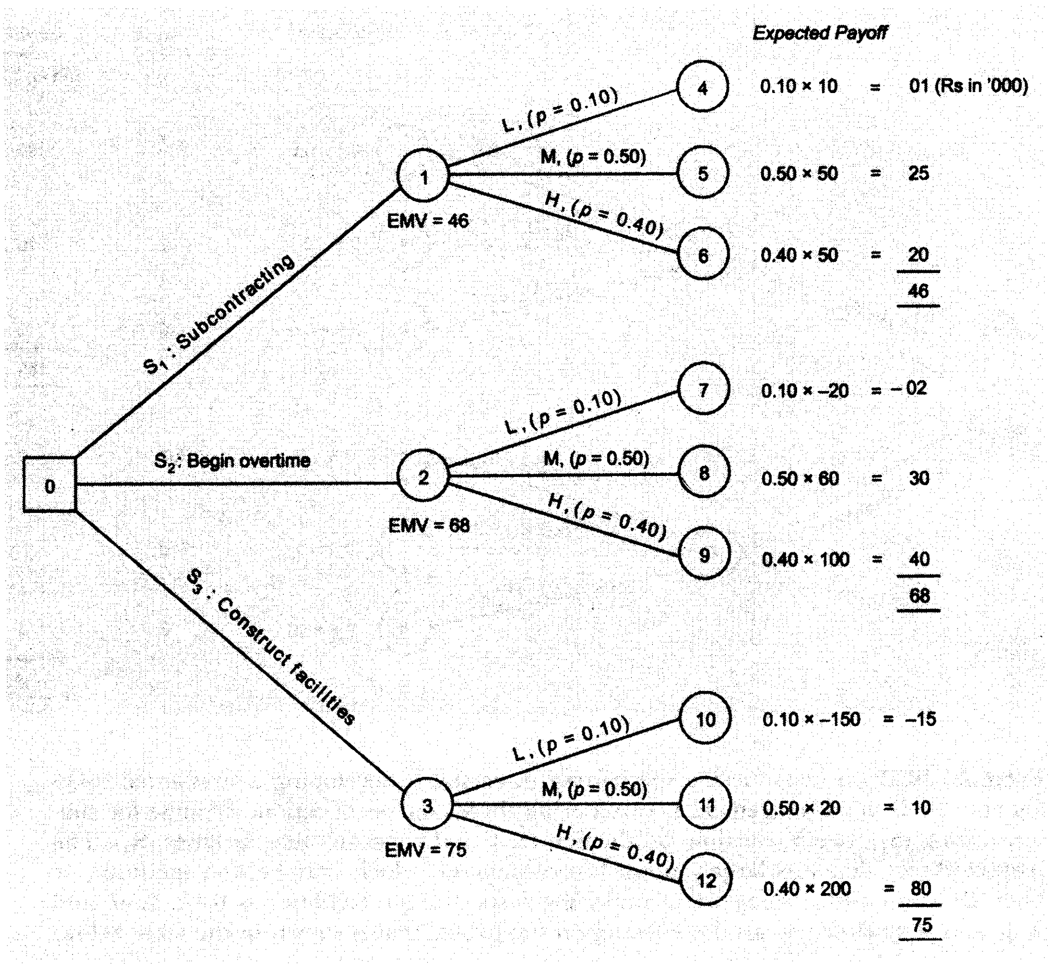
Solution: A decision tree which represents possible courses of action and states of nature are shown in Fig. 19.3. In order to analyze the tree, we start working backward from the end branches.

The most preferred decision at the decision node 0 is found by calculating the expected value of each decision branch and selecting the path (course of action) with high value.

Since node 3 has the highest EMV, therefore, the decision at node 0 will be to choose the course of action S_3, that is, construct new facilities.

Example 19.18: A businessman has two independent investment portfolios A and B available to him, but he lacks the capital to undertake both of them simultaneously. He can choose A first and then stop, or if A is not successful, then take, B or vice versa. The probability of success of A is 0.6, while for B it is 0.4. Both investment schemes require an initial capital outlay of Rs 10,000 and both return nothing if the venture is unsuccessful. Successful completion of A will return Rs 20,000 (over cost) and successful completion of B will return Rs 24,000 (over cost). Draw a decision tree and determine the best strategy. [*CA, May 1988; Delhi Univ., MBA, 1997*]

Figure 19.3
Decision Tree



Solution: The decision tree corresponding to the given information is depicted in the Fig. 19.4.

Table 19.33: Evaluation of Decision and Chance Nodes

Decision Point	*Outcome*	*Probability*	*Conditional Value (Rs)*	*Expected Value (Rs)*
D_3 (i) Accept A	Success	0.6	20,000	12000
	Failure	0.4	–10,000	– 4000
				8000
(ii) Stop	—	—	—	0
D_2 (i) Accept B	Success	0.4	24,000	9600
	Failure	0.6	–10,000	– 6000
				3600
(ii) Stop	—	—	—	0
D_1 (i) Accept A	Success	0.6	20,000 + 3600	14,160
	Failure	0.4	–10,000	– 4000
				10,160
(ii) Accept B	Success	0.4	24,000 + 8000	12,800
	Failure	0.6	–10,000	– 6000
				6800
(iii) Do nothing	—	—	—	0

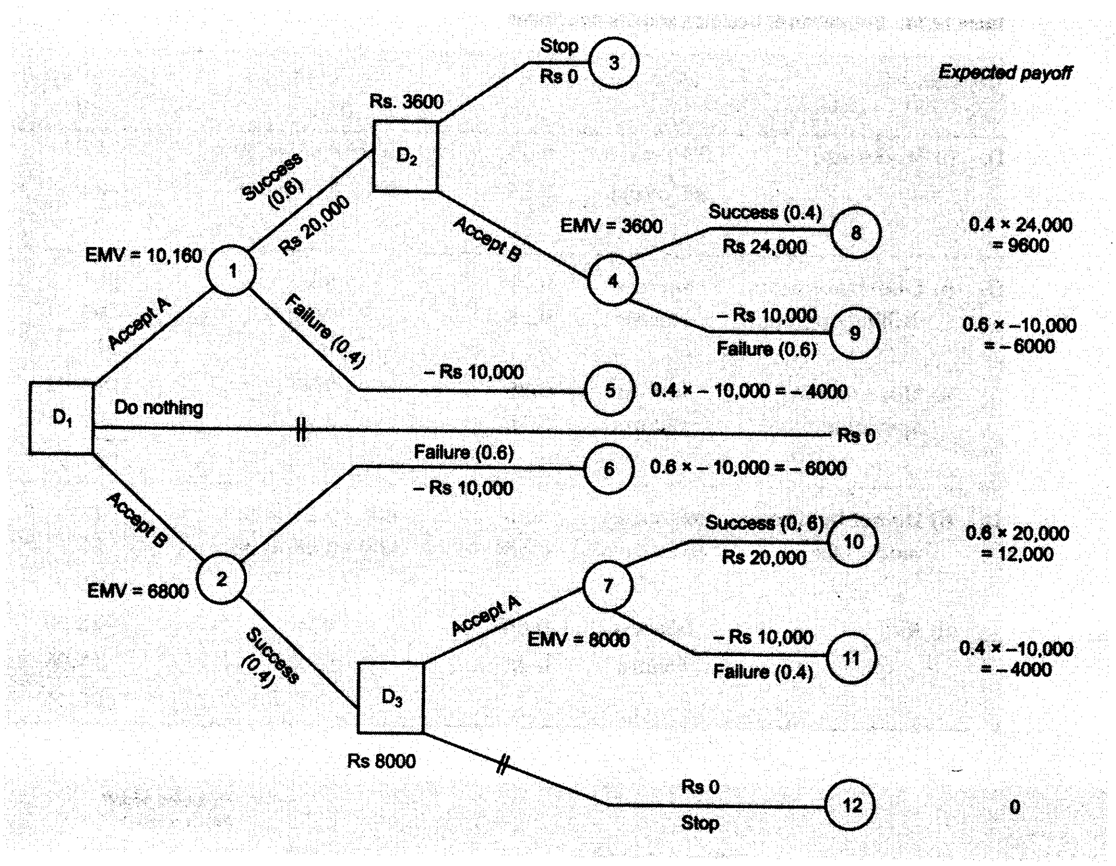


Figure 19.4
Decision Tree

Since the EMV = Rs 10,160 at node 1 is the highest, therefore, the best strategy at node D_1 is to accept course of action A first and if A is successful, then accept B.

Example 19.19: The Oil India Corporation (OIC) is considering whether to go for an offshore oil drilling contract to be awarded in the Bombay High. If OIC bids, the value would be Rs 600 million with a 65 per cent chance of gaining the contract. OIC may set up a new drilling operation or move their already existing operation, which has proved successful, to a new site. The probability of success and expected returns are as follows:

Outcome	*New Drilling Operation*		*Existing Operation*	
	Probability	*Expected Revenue (Rs in million)*	*Probability*	*Expected Revenue (Rs in million)*
Success	0.75	800	0.85	700
Failure	0.25	200	0.15	350

If the Corporation does not bid or lose the contract, they can use Rs 600 million to modernize their operations. This would result in a return of either 5 per cent or 8 per cent on the sum invested with probabilities 0.45 and 0.55. (Assume that all costs and revenue have been discounted to present value.)

(a) Construct a decision tree for the problem showing clearly the courses of action.

(b) By applying an appropriate decision criterion recommend whether or not the Oil India Corporation should bid for the contract.

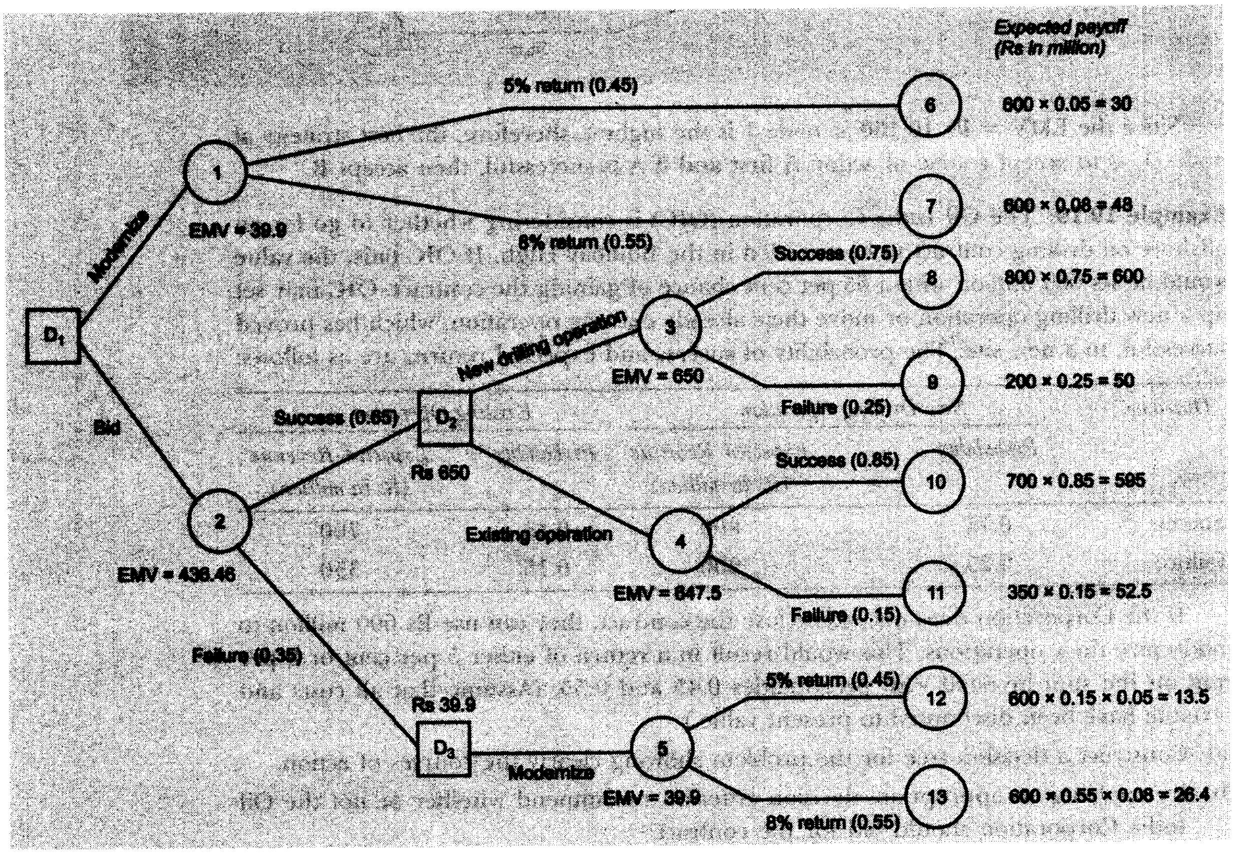
[*ICWA, June 1989*; *Delhi Univ., MBA, 1997, 2001*]

Solution: The decision tree based on the given information in the problem is depicted in the Fig. 19.5.

Table 19.34: Evaluation of Decision and Chance Nodes

[illegible]	Outcome	Probability	Conditional Value (Rs)	Expected Value (Rs)
D_3 (i) Modernize	5% return	0.45	600×0.05 = 30	13.5
	8% return	0.55	600×0.08 = 48	26.4
				39.9
D_2 (i) Undertake new drilling operation	Success	0.75	800	600
	Failure	0.25	200	50
				650
(ii) Move existing operation	Success	0.85	700	595
	Failure	0.15	350	52.5
				647.5
D_1 (i) Do not bid but modernize	5% return	0.45	600×0.05 = 30	13.5
	8% return	0.55	600×0.08 = 48	26.4
				39.9
(ii) Bid	Success	0.65	650	422.50
	Failure	0.35	39.9	13.96
				436.46

Figure 19.5
Decision Tree



Since the EMV = Rs 257.34 at node 2 is highest, therefore, the best decision at node D_2 is to bid and, if successful, establish a new drilling operation.

Example 19.20: Matrix Company is planning to launch a new product which can be introduced initially in Western India or in the whole country. If the product is introduced only in Western India, the investment outlay will be Rs 12 million. After two years, company can evaluate the project to determine whether it should cover the whole country. For such an expansion it will have to incur an additional investment of Rs 10 million. To introduce the product in the whole country right in the beginning would involve an outlay of Rs 20 million. The product, in any case, will have a life of 5 years, after which the plant will have zero net value.

If the product is introduced only in Western India, demand would be high or low with the probabilities of 0.8 and 0.2 respectively, and annual cash inflow of Rs 4 million and Rs 2.5 million, respectively.

If the product is introduced in the whole country right in the beginning the demand would be high or low with the probabilities of 0.6 and 0.4, respectively and annual cash inflows of Rs 8 million and Rs 5 million, respectively.

Based on the observed demand in Western India, if the product is introduced in the entire country the following probabilities would exist for high and low demand on an all-India basis:

Western India	*Whole Country*	
	High Demand	*Low Demand*
High demand	0.90	0.10
Low demand	0.40	0.60

The hurdle rate applicable to this project is 12 per cent.

(a) Set up a decision tree for the investment situation.

(b) Advise Matrix Company on the investment policy it should follow. Support your advice with appropriate reasoning. *[ICWA, June 1990]*

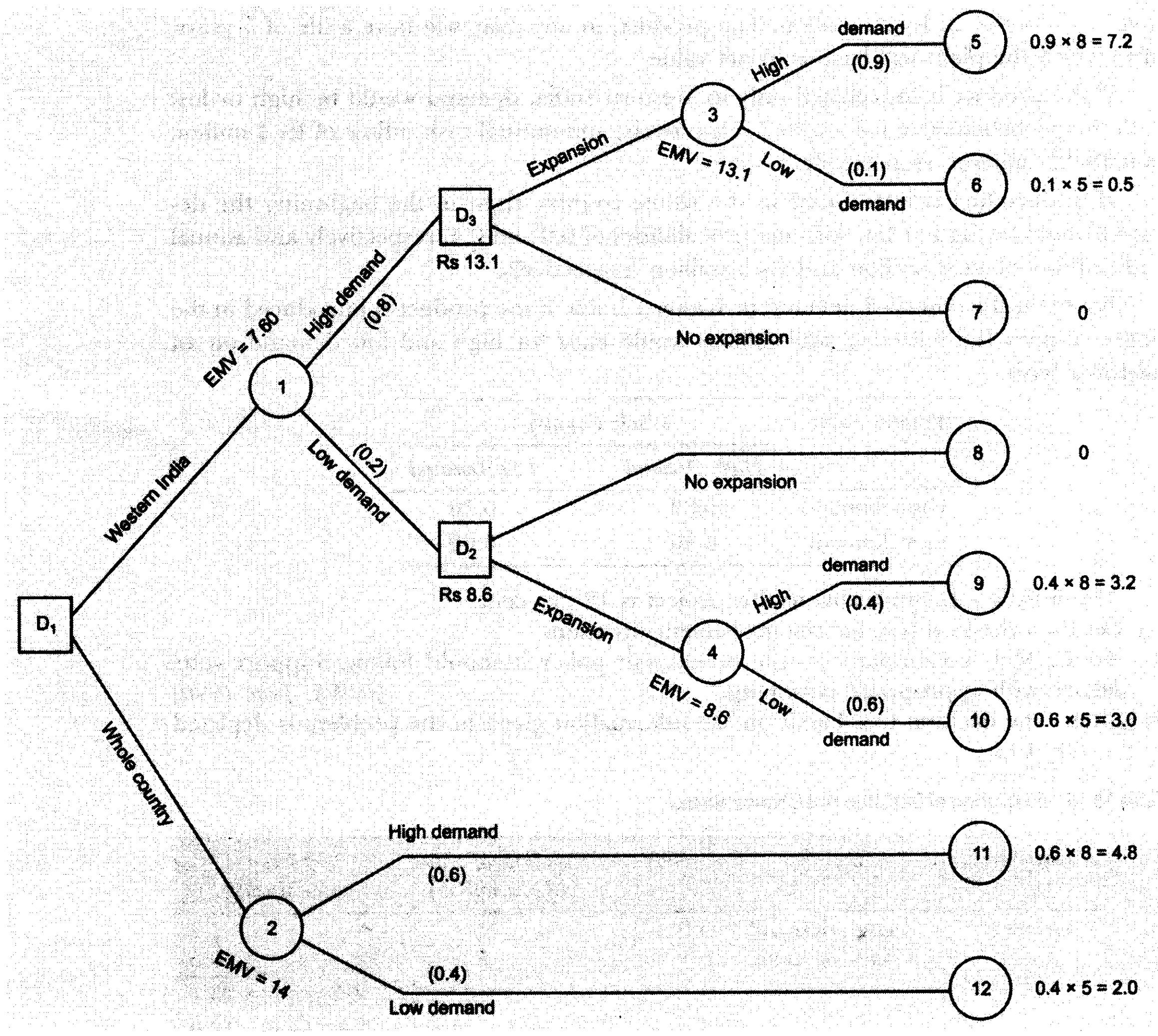
Solution: The decision tree based on the information given in the problem is depicted in the Fig. 19.6.

Table 19.35: Evaluation of Decision and Chance Nodes

Decision Point	*Outcome*	*Probability*	*Conditional Value (Rs in million)*	*Expected Value (Rs in million)*
D_3 (i) Expansion	High demand	0.9	8	7.2
	Low demand	0.1	5	0.5
				7.7 × 3 years = 23.1
(ii) Stop				Less: Cost 10.0
				13.1
				0
				13.1
D_2 (i) Expansion	High demand	0.4	8	3.2
	Low demand	0.6	5	3.0
				6.2 × 3 years = 18.6
				Less: Cost 10.0
				8.6
(ii) No Expansion				0
				8.6
D_1 (i) Introduction in whole country	High demand	0.6	8	4.8
	Low demand	0.4	5	2.0
				6.8 × 5 years = 34
				Less: Cost 20
				14
(ii) Introduction in Western India	High demand	0.8		(4 + 13.1) = 13.68
	Low demand	0.2		(2.5 + 8.6) = 2.22
				15.90
(iii) Do nothing				Less: Cost 12.00
				3.90

Since the EMV at node 2 is largest, make a decision to launch the product in the whole country.

Figure 19.6
Decision Tree



Conceptual Questions 19A

1. Discuss the difference between decision-making under certainty, uncertainty and risk.
2. What techniques are used to solve decision-making problems under uncertainty? Which technique results in an optimistic decision? Which technique results in a pessimistic decision?
3. Explain the various quantitative methods which are useful for decision-making under uncertainty.
4. What is a scientific decision-making process? Discuss the role of the statistical method in such a process.
5. Give an example of a good decision you made that resulted in a bad outcome. Also give an example of a good decision you made that had a good outcome. Why was each decision good or bad?
6. Given the complete set of outcomes in a certain situation, how is the EMV determined for a specific course of action? Explain in your own words.
7. Explain the difference between expected opportunity loss and expected value of perfect information.
8. Indicate the difference between decision-making under risk, and uncertainty in statistical decision theory.
9. Briefly explain 'expected value of perfect information' with examples. [*CA, May 1992*]
10. Describe a business situation where a decision-maker faces a decision under uncertainty and where a decision based on maximizing the expected monetary value cannot be made. How do you think the decision-maker should make the required decision?

Self-Practice Problems 19C

19.11 Raman Industries Ltd. has a new product which they expect has great potential. At the moment they have two courses of action open to them: To test market (S_1) or to drop the product (S_2).

If they test it, it will cost Rs 50,000 and the response could be positive or negative with probabilities 0.70 and 0.30, respectively. If it is positive, they could either market it with full effort or drop the product. If they market on a full scale, then the result might be low, medium, or high demand and the respective net payoffs would be—Rs 1,00,000, Rs 1,00,000, or Rs 5,00,000. These outcomes have probabilities of 0.25, 0.55, and 0.20, respectively. If the result of the test marketing is negative they may decide to drop the product. If, at any point, they drop the product there is a net gain of Rs 25,000 from the sale of scrap. All financial values have been discounted to the present. Draw a decision tree for the problem and indicate the most preferred decision.

19.12 A manufacturing company has just developed a new product. On the basis of past experience, a product such as this will either be successful, with an expected gross return of Rs 1,00,000, or unsuccessful, with an expected gross return of Rs 20,000. Similar products manufactured by the company have a record of being successful about 50 per cent of the time. The production and marketing costs of the new product are expected to be Rs 50,000.

The company is considering whether to market this new product or to drop it. Before making its decision, a test marketing effort can be conducted at a cost of Rs 10,000. Based on past experience, test marketing results have been favourable about 70 per cent of the time. Furthermore, products favourably tested have been successful 80 per cent of the time. However, when the test marketing result has been unfavourable, the product has been successful only 30 per cent of the time. What course of action should the company pursue?

19.13 The XYZ Company manufactures guaranteed tennis balls. At present time, approximately 10 per cent of the tennis balls are defective. A defective ball leaving the factory costs the company Re 0.50 to honour its guarantee. Assume that all defective balls are returned. At a cost of Re 0.10 per ball, the company can conduct a test, which always correctly identifies both good and bad tennis balls.

(a) Draw a decision tree and determine the optimal course of action and its expected cost.

(b) At what testing cost should the company be indifferent to testing?

19.14 The Ore Mining Company is attempting to decide whether or not a certain piece of land should be purchased. The land cost is Rs 3,00,000. If there are commercial ore deposits on the land, the estimated value of the property is Rs 5,00,000. If no ore deposits exist, however, the property value is estimated at Rs 2,00,000. Before purchasing the land, the property can be cored at a cost of Rs 20,000. The coring will indicate if conditions are favourable or unfavourable for ore mining. If the coring report is favourable the probability of recoverable ore deposits on the land is 0.8, while if the coring report is unfavourable the probability is only 0.2. Prior to obtaining any coring information, management estimates that the odds are 50–50 that ore is present on the land. Management has also received coring reports on pieces of land similar to the one in question and found that 60 per cent of the coring reports were favourable. Construct a decision tree and determine whether the company should purchase the land, decline to purchase it, or take a coring test before making its decision. Specify the optimal course of action and the EMV.

19.15 XYZ company, dealing with a newly invented telephone device, is faced with the problem of selecting one of the following courses of action available:

(a) manufacture the device itself; or

(b) be paid on a royalty basis by another manufacturer; or

(c) sell the rights for its invention for a lump sum.

The profit (Rs in '000s) which can be expected in each case and the probabilities associated with the level of sales are shown in the following table:

Outcome	*Prob*	*Manufacture Itself*	*Royalties*	*Sell all Rights*
High sales	0.1	75	35	15
Medium sales	0.3	25	20	15
Low sales	0.6	– 10	10	15

Represent the company's problem in the form of a decision tree. Redraw the decision tree further by introducing the following additional information:

(a) If it manufactures itself and sales are medium or high, then company has the opportunity of developing a new version of the telephone;

(b) From past experience the company estimates that there is a 50 per cent chance of successful development,

(c) The cost of development is Rs 15 and returns after deduction of development cost are Rs 30 and Rs 10 for high and medium sales, respectively.

19.16 An automobile owner faces the decision as to what deductible amount of comprehensive insurance coverage to select. Comprehensive coverage includes losses due to fire, vehicle theft, vandalism, and the forces of nature. The possible choices are zero deductible coverage for Rs 60 per year or Rs 50 deductible coverage for Rs 45 per year. (The owner pays the first Rs 50 of any loss of at least Rs 500.) Considering accidents covered by the comprehensive portion of the policy, some of the owner's utility values are given below:

Amount (Rs) :	−95	−60	−50	−45	0
Utility :	0.2	0.4	0.45	0.47	0.5

(a) Sketch the owner's utility curve. Is the owner a risk avoider, an EMV'er, or a risk taker?

(b) On the basis of the utility curve drawn in part (a), should the owner take the zero deductible or the Rs 50 deductible comprehensive coverage?

Hints and Answers

19.11 The company should test market rather than drop the product. If the test market result is positive, the company should market the product otherwise drop it.

19.12 Test marketing should be done; EMV = Rs 13,800.

19.13 (a) Do not test, Rs 0.05; (b) Rs 0.05

19.14 Test: If favourable—purchase; If not—do not purchase; Rs 64,000

19.15 Royalties; IMV = Rs 15.5
If p is the probability of high sale, then
(a) manufacture itself when $p > 0.24$
(b) pay on royalty basis when $0.07 < p < 0.24$
(c) sell all rights for lump sum when $p < 0.071$

19.16 (b) Rs 50 deductible.

Formulae Used

1. **Expected monetary value** $EMV(S_j) = \sum_{i=1}^{m} p_{ij} p_i$

   p_i = probability of occurrence of state of nature i $(i = 1, 2, \ldots, m)$

   p_{ij} = payoff associated with state of nature, N_i and decision alternative, S_j

2. **Expected opportunity loss** $EOL(N_i) = \sum_{i=1}^{m} l_{ij} p_i$

   l_{ij} = opportunity loss due to state of nature N_i and decision alternative S_j

3. **Expected value of perfect information**

   $$EVPI = \sum_{i=1}^{m} p_{ij}(N_i) - \text{Max EMV}$$

4. **Expected profit with perfect information**

   $$EPPI = EVPI + EMV$$

5. **Baye's rule** $P(A_i \mid B) = \dfrac{P(A_i)\,P(B|A_i)}{\sum P(A_i)\,P(B|A_i)}$ for all i

Chapter Concepts Quiz

True or False

1. Classical methods of statistical inference are based on the use of sample data and the objective interpretation of probabilities. (T/F)
2. Techniques of statistical decision analysis have been developed with continuous as well as discrete variables. (T/F)
3. The expected payoff criterion is based on the use of probability values as well as economic consequences. (T/F)
4. A payoff table cannot include the probability value for each event. (T/F)
5. Acts are referred to as the strategies available to the decision-maker. (T/F)
6. The expected payoff criterion identifies the best decision alternative for which the payoff is the highest. (T/F)
7. The minimum EOL value is also the act that has the maximum expected payoff. (T/F)
8. EVPI = EP (with perfect information) + EP (under uncertainty). (T/F)
9. EVPI ≠ EOL (best act under uncertainty). (T/F)
10. Maximin criterion is reversed if the consequences are in terms of opportunity loss. (T/F)

Multiple Choice

11. A type of decision-making environment is:
    (a) certainty (b) uncertainty
    (c) risk (d) all of the above
12. Decision theory is concerned with:
    (a) methods of arriving at an optimal decision
    (b) selecting an optimal decision in sequential manner
    (c) analysis of information that is available
    (d) all of the above

13. Which of the following criteria is not used for decision-making under uncertainty?
(a) maximin (b) maximax
(c) minimax (d) minimize expected loss

14. Which of the following criteria is not applicable to decision-making under risk?
(a) maximize expected return
(b) maximize return
(c) minimize expected regret
(d) knowledge of likelihood occurrence of each state of nature

15. The minimum expected opportunity loss (EOL) is:
(a) equal to EVPI (b) minimum regret
(c) equal to EMV (d) both (a) and (b)

16. The expected value of perfect information (EVPI) is:
(a) equal to the expected regret of the optimal decision under risk
(b) the utility of additional information
(c) the maximum expected opportunity loss
(d) none of the above

17. The value of the coefficient of optimism (α) is needed while using the criterion of:
(a) equally likely (b) maximin
(c) realism (d) minimax

18. The decision-maker's knowledge and experience may influence the decision-making process when using the criterion of:
(a) maximax (b) minimax regret
(c) realism (d) maximin

19. The difference between the expected profit under conditions of risk and the expected profit with perfect information is called:
(a) expected value of perfect information
(b) expected marginal loss
(c) expected opportunity loss
(d) none of the above

20. The concept of utility is used to:
(a) measure the utility of money
(b) take into account aversion of risk
(c) both (a) and (b)
(d) none of the above

Concepts Quiz Answers

1. T	2. T	3. T	4. F	5. T	6. T	7. T	8. F	9. F
10. T	11. (d)	12. (d)	13. (d)	14. (b)	15. (d)	16. (a)	17. (c)	18. (c)
19. (a)	20. (c)							

Review Self-Practice Problems

19.17 A producer of boats has estimated the following distribution of demand for a particular kind of boat:

No. demanded :	0	1	2	3	4	5	6
Probability :	0.14	0.27	0.27	0.18	0.09	0.04	0.01

Each boat costs him Rs 7000 and he sells them for Rs 10,000 each. Boats left unsold at the end of the season must be disposed of for Rs 6000 each. How many boats should be in stock so as to maximize his expected profit? *[Delhi Univ., MCom, 1988]*

19.18 A small industry finds from past data that the cost of making an item is Rs 25, the selling price is Rs 30 if it is sold within a week, and it could be disposed of at Rs 20 per item at the end of the week:

Weekly sales :	< 3	4	5	6	7	≥ 8
No. of weeks :	0	10	20	40	30	0

Find the optimum number of items per week the industry should produce. *[CA, May 1986]*

19.19 A firm makes pastries which it sells at Rs 8 per dozen in special boxes containing one dozen each. The direct cost of pastries for the firm is Rs 4.50 per dozen. At the end of the week the stale pastries are sold off at a lower price of Rs 3.50 per dozen. The overhead expenses attributable to pastry production are Rs 1.25 per dozen. Fresh pastries are sold in special boxes which cost 50 paise each and the stale pastries are sold wrapped in ordinary paper. The probability distribution of demand per week is as under:

Demand (in dozen) :	0	1	2	3	4	5
Probability :	0.01	0.14	0.2	0.5	0.1	0.05

Find the optimal production level of pastries per week. *[Delhi Univ., MCom, 1986]*

19.20 The local football club wants your advice on the number of programmes that should be printed for each game. The cost of printing and production of programmes for each game, as quoted by the local printer, is Rs 1000 plus 4 paise per copy. Advertising revenue which has been agreed upon for the season represents Rs 800 for each game.

Programmes are sold for 15 paise each. A review of sales during the previous season indicates that the following pattern is expected to be repeated during the coming season of 50 games:

Number of Programmes Sold	*Number of Games*
10,000	5
20,000	20
30,000	15
40,000	10

Programmes not sold at the game are sold as waste paper to a paper manufacturer at one paisa per copy.

Assuming that the four options listed are the only possibilities,

(a) prepare a payoff table;

(b) determine the number of programmes that would provide the largest profit, if a constant number of programmes were to be printed for each game;

(c) calculate the profit which would arise from a perfect forecast of the number of programmes which would be sold at each game.

19.21 The probability distribution of monthly sales (units) of an item is as follows:

Monthly sales :	0	1	2	3	4	5	6
Probability :	0.01	0.06	0.25	0.30	0.22	0.10	0.06

The cost of carrying inventory (unsold during the month) is Rs 30 per unit per month and the cost of unit shortage is Rs 70. Determine the optimum stock to minimize expected cost. *[CA, May 1987]*

19.22 A modern home appliances dealer finds that the cost of holding a mini cooking range in stock for a month is Rs 200 (insurance, minor deterioration, interest on borrowed capital, etc.). Customers who cannot obtain a cooking range immediately tend to go to other dealers and he estimates that for every customer who cannot get immediate delivery, he loses an average of Rs 500. The probabilities of a demand of 0, 1, 2, 3, 4, 5 mini cooking ranges in a month are 0.05, 0.10, 0.20, 0.30, 0.20, and 0.15 respectively. Determine the optimal stock level of cooking ranges. Also find the EVPI. *[Delhi Univ., MBA, 1990]*

19.23 A TV dealer finds that the cost of holding a TV in stock for a week is Rs 50. Customers who cannot obtain new TVs immediately tend to go to other dealers and he estimates that for every customer who cannot get immediate delivery he loses an average of Rs 200. For one particular model of TV the probabilities of a demand of 0, 1, 2, 3, 4 and 5 TVs in a week are 0.05, 0.10, 0.20, 0.30, 0.20, and 0.15, respectively.

(a) How many televisions per week should the dealer order? Assume there is no time lag between ordering and delivery.

(b) Compute the EVPI.

(c) The dealer is thinking of spending on a small market survey to obtain additional information regarding demand levels. How much should he be willing to spend on such a survey? *[Delhi Univ., MBA, 1988]*

19.24 XYZ Co. Ltd wants to go in for a public share issue of Rs 10 lakh (1 lakh shares of Rs 10 each) as a part of its efforts to raise capital needed for its expansion programme. The company is optimistic that if the issue were made now it would be fully taken up at a price of Rs 30 per share.

However, the company is facing two crucial situations both of which may influence the share prices in the near future. These are:

(a) An impending wage dispute with assembly workers which could lead to a strike in the whole factory and could have an adverse effect on the share price.

(b) The possibility of substantial business in the export market which would increase the share price.

The four possible events and their expected effect on the company's share prices are envisaged as:

E_1 : No strike, and no export business obtained—share price stays at Rs 34.

E_2 : Strike, and export business obtained—share price stays at Rs 30.

E_3 : No strike, and export business lost—share price hovers around Rs 32.

E_4 : Strike, and export business lost—share price drops to Rs 16.

And the management has identified three possible strategies that the company could adopt, namely

S_1 : Issue 1,00,000 shares now.

S_2 : Issue 1,00,000 shares only after the outcome of (i) and (ii) above are known.

S_3 : Issue 50,000 shares now and 50,000 shares after the outcome (i) and (ii) are known.

You are required to:

(a) Draw up a payoff table for the company and determine the minimax regret solution. What alternative criteria might be used?

(b) Determine the optimum policy for the company using the criterion of maximizing expected payoff, given the estimate that the probability of a strike is 55 per cent and there is a 65 per cent chance of getting the export business, these probabilities being independent.

(c) Determine the expected value of perfect information for the company. *[ICWA, June 1989]*

19.25 A company manufacturing large electrical equipment is anticipating the possibility of a total or a partial copper strike in the near future. It is attempting to decide whether to stockpile a large amount of copper, at an additional cost of Rs 50,000; a small amount, costing an additional Rs 20,000: or to stockpile no additional copper at all. The stockpiling costs, consisting of excess storage, holding and handling costs and so forth, are over and above the actual material costs.

If there is a partial strike, the company estimates that an additional cost of Rs 50,000 for delayed orders will be incurred if there is no stockpile at all. If a total strike occurs, the cost of delayed orders is estimated at Rs 1,00,000 if there is only a small stockpile and Rs 2,00,000 with no stockpile. The company estimates the probability of a total strike as 0.1 and that of a partial strike as 0.3.

(a) Develop a conditional cost table showing the cost of all outcomes and course of action combinations.

(b) Determine the preferred course of action and its cost. What is the EPPI?

(c) Develop a conditional opportunity loss table.

(d) Without calculating the EOL, find the EVPI.

19.26 A well-known departmental store advertises female fashion garments from time to time in the Sunday press. Only one garment is advertised on each occasion. Experience of garments in the price range Rs 30–40 leads the management to assess the statistical probability of demand at various levels after each advertisement as follows:

Demand (No. of garments)	*After Advertising in One Newspaper*	*After Advertising in Two Newspapers*
30	0.10	0.00
40	0.25	0.15
50	0.40	0.35
60	0.25	0.40
70	0.00	0.10

The next garment to be advertised at Rs 35 will cost Rs 15 to make and can be disposed of to the trade, if unsold, for Rs 10. The store's advertising agents will charge Rs 37.50 for artwork and blockmaking for the advertisement and each newspaper will charge Rs 50 for the insertion of the advertisement. You are required to:

(a) Calculate how many garments to the nearest 10 should be purchased by the store in order to maximize expected gross profit, after advertising in: (i) one newspaper (ii) two newspapers.

(b) Calculate the amount, if any, of the expected net profit after advertising: (i) in one newspaper (ii) two newspapers.

19.27 A car manufacturer uses a special control device in each car he produces. Two alternative methods can be used to detect and avoid a faulty device. Under the first method, each device is tested before it is installed. The cost is Rs 2 per test. Alternatively, the control device can be installed without being tested, and a faulty device can be detected and rendered after the car has been assembled, at a cost of Rs 20 per faulty device.

Regardless of which method is used, faulty devices cannot be repaired and must be discarded.

A manufacturer purchases the control devices in batches of 10,000. Based on past experience, he estimates the proportion of defective components and the associated probability to be:

Proportion of Faulty Devices	*Probability*
0.08	0.20
0.12	0.70
0.16	0.10

(a) Which inspection method should the manufacturer adopt?

(b) What is the expected value of perfect information (EVPI)?

19.28 Suppose a company has several independent investment opportunities each of which has an equal chance of gaining Rs 1,00,000 or losing Rs 60,000. What is the probability that the company will lose money on two such investments? On three such investments? On four such investments?

If a company has a number of independent investment opportunities, in each of which the financial risk is relatively small compared to its overall asset position, why should the company try to maximize EMV, rather than expected utility?

19.29 An oil drilling company is considering the purchase of mineral rights on a property of Rs 100 lakh. The price includes tests to indicate whether the property has type A geological formation or type B geological formation. The company will be unable to tell the type of geological formation until the purchase is made. It is known, however, that 40 per cent of the land in this area has type A formation and 60 per cent type B formation. If the company decides to drill on the land it will cost Rs 200 lakh. If the company does drill it may hit an oil well, gas well or a dry hole. Drilling experience indicates that the probability of striking an oil well is 0.4 on type A and 0.1 on type B formation. Probability of hitting gas is 0.2 on type A and 0.3 on type B formation. The estimated discounted cash value from an oil well is Rs 1000 lakh and from a gas well is Rs 500 lakh. This includes everything except cost of mineral rights and cost of drilling. Use the decision-tree approach and recommend whether the company should purchase the mineral rights? *[ICWA, June 1987]*

19.30 The Sensual Cosmetic Co. has developed a new perfume which management feels has a tremendous potential. It not only interacts with the wearer's body chemistry to create a unique fragrance but is also especially long-lasting. A total of Rs 10 lakh has already been spent on its development. Two marketing plans have been devised:

(a) The first plan follows the company's usual policy of giving small samples of the new products when other items in the company's product lines are purchased and placing advertisements in women's magazines. The plan would cost Rs 5 lakh and it is believed that it might result in a high, moderate, or low market response with probability of 0.2, 0.5, and 0.3, respectively. The net profit excluding development and promotion costs in these cases would be Rs 20 lakh, Rs 10 lakh, and Rs 1 lakh, respectively. If it later appears that the market response is going to be low it would be possible to launch a TV advt. campaign. This would cost another Rs 7.5 lakh. It would change the market response to high or moderate as previously described but with probability of 0.5 each.

(b) The second marketing plan is much more aggressive than the first. The emphasis would be heavily upon TV advertising. The total cost of this plan would be Rs 15 lakh, but the market response would be either excellent or good, with probabilities of 0.4 and 0.6, respectively. The profit excluding development and promotion costs would be Rs 30 lakh and Rs 25 lakh for the two outcomes.

Advise on the sequence of strategy to be followed by the company. *[ICWA, Dec. 1987]*

19.31 The investment staff of TNC Bank is considering four investment proposals for a client: shares, bonds, real estate and saving certificates. These investments will be held for one year. The past data regarding the four proposals are given below:

Shares: There is a 25 per cent chance that shares will decline by 10 per cent, a 30 per cent chance that they will remain stable and a 45 per cent chance that they will increase in value by 15 per cent. Also the shares under consideration do not pay any dividends.

Bonds: These bonds stand a 40 per cent chance of increase in value by 5 per cent and 60 per cent chance of remaining stable and yield 12 per cent.

Real estate: This proposal has a 20 per cent chance of increasing 30 per cent in value, a 25 per cent chance of increasing 20 per cent in value, a 40 per cent chance of increasing 10 per cent in value, a 10 per cent chance of remaining stable and a 5 per cent chance of losing 5 per cent of its value.

Savings certificates: These certificates yield 8.5 per cent with certainty.

Use a decision tree to structure the alternatives to the investment staff, and using the expected value criterion, choose the alternative with the highest expected value. [*CA, Nov. 1990*]

19.32 The corporation has recently got leasehold drilling rights on a large area in the western part of the country. No seismic coverage is available and to conduct a detailed survey Rs 3 million is required. If oil is struck, a large reserve may result in a net profit of Rs 30 million, whereas a smaller marginal reserve may result in a net profit of Rs 18 million. Cost of drilling wildcast well is Rs 7 million.

Seismic is thought to be quite reliable in this area. Uncertainty involved is whether or not a structure exists. The company assesses probability that the test producing good, fair or bad result is 0.40, 0.30 and 0.30, respectively. On the basis of past drilling records and experiences indicating the probabilities of striking oil in large reserve, smaller marginal reserve or dry hole even in the presence of good, fair and bad reading of seismic study are as under:

Seismic Study	*Probability of Yield*		
	Large Reserve	*Marginal Reserve*	*Dry Hole*
Good	0.50	0.25	0.25
Fair	0.30	0.30	0.40
Bad	0.10	0.20	0.70

Exploratory group has suggested two possible exploration strategies:

(a) Drill at once on the basis of present geologic intepretation and extrapolations.

(b) Conduct a seismic study and defer drilling till seismic data is reviewed.

As a member of strategic group of the company evaluate the two strategies suggested by the exploratory group.

19.33 A company has developed a new product in its R&D laboratory. The company has the option of setting up production facility to market this product straightaway. If the product is successful, then over the three years expected product life, the returns will be Rs 120 lakh with a probability of 0.70. If the market does not respond favourably, then the returns will be only Rs 15 lakh with probability of 0.30.

The company is considering whether it should test market this product building a small pilot plant. The chance that the test market will yield favourable response is 0.80. If the test market gives favourable response, then the chance of successful total market improves to 0.85.

If the test market gives poor response, then the chance of success in the total market is only 0.30.

As before, the returns from a successful market will be Rs 120 lakh and from an unsuccessful market only Rs 15 lakh. The installation cost to production is Rs 40 lakh and the cost of test marketing pilot plant is Rs 5 lakh. Using decision-tree analysis, draw a decision tree diagram, carry out necessary analysis to determine the optimal decisions. [*Delhi Univ., MBA, 1999*]

Hints and Answers

19.17 Conditional profit value

= MP × boats sold – ML × boats unsold

= (10,000 – 7,000) boats sold – (7,000 – 6,000) boats unsold

$$= \begin{cases} 300\,D & ; \quad D \geq S \\ 3000\,D - 1000\,(S - D) = 4000D - 1000S; & D < S \end{cases}$$

where D is the number of boats sold and S is the number of boats produced

EMV* = 4,080, stock 3 boats.

19.18 Conditional profit value

= MP × item sold – ML × item unsold

= (30 – 25) item sold – (25 – 20) item unsold

$$= \begin{cases} (30-25)\,D & ; \quad D \geq S \\ (30-25)\,D - (25-20)\,(S-D) = 10D - 5S; & D < S \end{cases}$$

where, D is the number of item sold; S is the number of item produced.

EMV = Rs 26, produce 6 items.

19.19 Conditional profit value

= MP × boxes sold – ML × boxes unsold

= (8 – 4.50 – 1.25 – 0.50) boxes sold – (4.50 + 1.125 – 5.50) boxes unsold

$$= \begin{cases} 1.75\,D & ; \quad D \geq S \\ 1.75\,D - 0.25\,(S - D) = 2\,D - 0.25\,S; & D < S \end{cases}$$

EMV* = Rs 4.28, produce 4 dozen pastries.

19.20 (a) Conditional profit = Sales revenue – Cost

= {0.15D + 800 + (P – D) × 0.101} – (1,000 + 0.04P)

$$= \begin{cases} 0.14\,D - 0.03\,P - 20; & D < P \\ 0.11\,D - 200 & ; \quad D \geq P \end{cases}$$

where P is the number of programmes printed and D is the demand for programmes.

(b) largest expected profit = Rs 2260 accrues for P = 30,000 copies per game

(c) In case of perfect forecast, expected profit is:
900 × 0.1 + 2000 × 0.4 + 3,100 × 0.3 + 4200 × 0.2 = Rs 2660
EVPI = Rs 2660 – Rs 2260 = Rs 400

19.21 Cost function $= \begin{cases} 70\,(D-S)\ ; & D \geq S \\ 200\,(S-D); & D < S \end{cases}$

where D = monthly demand (or sales); S = number of units purchased.

Since expected cost Rs 46 is minimum for course of action 4, optimum stock to minimize the cost is 4 units per month.

19.22 Cost function $= \begin{cases} 500\,(D-S); & D \geq S \\ 200\,(S-D); & D < S \end{cases}$

Since expected cost Rs 315 is minimum for course of action 4, optimum stock to minimize the cost is 4 cooking ranges.

19.23 Cost function $= \begin{cases} 50\,S + 200\,(D-S); & D \geq S \\ 50\,D + 50\,(S-D); & D < S \end{cases}$

where D = number of TV sets demanded; S = number of TV sets ordered (or stored)

Since expected cost Rs 230 is minimum, therefore, dealer must order 4 TV sets per week.

EVPI = Minimum expected cost – Expected cost under perfect information = 230 – 147.5 = Rs 82.5

19.24 (a) S_1; Rs 4; Alternative: Maximum criterion; S_1, Rs 30.

(b) Adopt S_1; Rs 30.075 lakh

(c) EVPI = EPPI – Optimal EMV = 31.48 – 30.075 = Rs 1.40 lakh.

19.25 (b) small stockpile; Rs 30,000 – Rs 11,000; (d) Rs 19,000

19.26 Marginal profit (MP) = Rs (35 – 15) = Rs 20
Marginal loss (ML) = Rs (15 – 10) = Rs 5
Conditional payoff = MP × garments sold – ML × garments unsold

(a) EMV (S) = Rs 600; EMV (S_2) = Rs 775;
EMV (S_3) = Rs 887.5;
EMV (S_4) = Rs 900; EMV (S_5) = Rs 850
Max. EMV is corresponding to course of action S_4, i.e. purchase 60 garments.

(b) EMV (S_2) = Rs 800;
EMV (S_3) = Rs 962.5; EMV (S_4) = Rs 1,037.5;
EMV (S_5) = Rs 1,012.5.
Max. EMV is corresponding to course of action S_4, i.e. purchase 60 garments.

19.27 (a) First alternative, Rs 20,000

(b) EVPI = Expected cost under uncertainty – Expected cost with perfect information
= 20,000 – (16,000 × 0.2 + 20,000 × 0.7 + 20,000 × 0.1)
= 20,000 – 19,200 = Rs 800 per batch.

19.28 0.25; 0.50; 0.3125

19.29

Decision Point	*Outcome*	*Probability*	*Conditional Value (Rs)*	*Expected Value (Rs)*
D_3 (i) Drill	Oil well	0.1	1000	100
	Dry hole	0.6	0	0
	Gas	0.3	500	150
				250
			Less: Cost	200
				50
(ii) Do not drill				0
				Total 50
D_2 (i) Drill	Oil well	0.4	1,000	400
	Dry hole	0.4	0	0
	Gas	0.2	500	100
				500
			Less: Cost	200
				300
(ii) Do not drill				0
			Total	300
D_1 (i) Type A formation		0.4	300	120
(ii) Type B formation		0.6	50	30
				150
			Less: Cost	100
			Total	50

The company should purchase the mineral rights.

19.30 Aggressive Plan.

Appendices

Table A1: Poission Probabilities

x	*0.005*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	[illegible]	[illegible]
0	.9950	.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
1	.0050	.0099	.0196	.0291	.0384	.0476	.0565	.0653	.0738	.0823
2	.0000	.0000	.0002	.0004	.0008	.0012	.0017	.0023	.0030	.0037
3	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001
x	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
x	*1.1*	*1.2*	*1.3*	*1.4*	*1.5*	*1.6*	*1.7*	*1.8*	*1.9*	*2.0*
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707
3	.0738	.0867	.0998	.1128	.1155	.1378	.1496	.1607	.1710	.1804
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034
8	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002

Contd.

x	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1496
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	.1680
5	.0417	.0476	.0538	.0602	.0668	.0735	.8004	.0872	.0940	.1008
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002

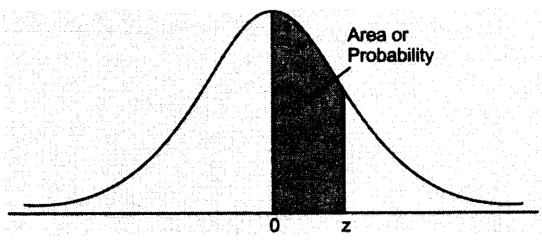
x	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247	.0224	.0202	.0183
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915	.0850	.0789	.0733
2	.2165	.2087	.2008	.1929	.1850	.1771	.1692	.1615	.1539	.1459
3	.2237	.2226	.2209	.2186	.2158	.2125	.2987	.2046	.2001	.1954
4	.1733	.1781	.1823	.1858	.1888	.1912	.1931	.1944	.1951	.1954
5	.1075	.1140	.1203	.1265	.1322	.1377	.1429	.1477	.1522	.1563
6	.0555	.0608	.0662	.0716	.0771	.0826	.0881	.0936	.0989	.1042
7	.0246	.0278	.0312	.0348	.0385	.0425	.0466	.0508	.0551	.0595
8	.0095	.0111	.0129	.0148	.0169	.0191	.0225	.0241	.0269	.0298
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089	.0102	.0116	.0132
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033	.0039	.0045	.0053
11	.0003	.0004	.0005	.0006	.0007	.0009	.0011	.0013	.0016	.0019
12	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006
13	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.00001

x	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	.0166	.0150	.0136	.0123	.0111	.0101	.0191	.0082	.0074	.0067
1	.0679	.0630	.0583	.0540	.0500	.0462	.0427	.0395	.0365	.0337
2	.1393	.1323	.1254	.1188	.1125	.1063	.1005	.0948	.0894	.0842
3	.1904	.1852	.1798	.1743	.1687	.1631	.1574	.1517	.1460	.1404
4	.1951	.1944	.1933	.1917	.1898	.1875	.1849	.1820	.1789	.1755
5	.1600	.1633	.1662	.1687	.1708	.1725	.1738	.1747	.1753	.1755
6	.1093	.1143	.1191	.1237	.1281	.1323	.1362	.1398	.1432	.1462
7	.0640	.0686	.0732	.0778	.0824	.0869	.0914	.0959	.1002	.1044
8	.0328	.0360	.0393	.0428	.0463	.0500	.0537	.0575	.0614	.0653
9	.0150	.0168	.0188	.0209	.0232	.0255	.0281	.0307	.0334	.0363
10	.0061	.0071	.0081	.0092	.0104	.0118	.0132	.0147	.0164	.0181
11	.0023	.0027	.0032	.0037	.0043	.0049	.0056	.0064	.0073	.0082
12	.0008	.0009	.0011	.0013	.0016	.0019	.0022	.0026	.0030	.0034
13	.0002	.0003	.0004	.0005	.0006	.0007	.0008	.0009	.0011	.0013
14	.0001	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005
15	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002

Table A2: Binomial Coefficients

n	$^nC_0$	$^nC_1$	$^nC_2$	$^nC_3$	$^nC_4$	$^nC_5$	$^nC_6$	$^nC_7$	$^nC_8$	$^nC_9$	$^nC_{10}$
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	3005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

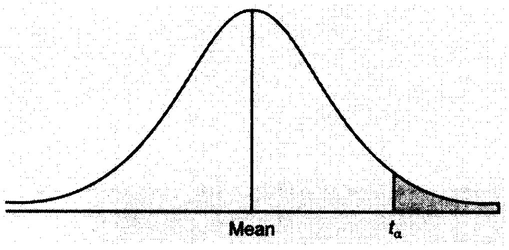
Table A3: Area of Standard Normal Distribution



Areas under the standard normal probability distribution between normal variate $z = 0$ and a positive value of z. Areas for negative value of z are obtained by symmetry.

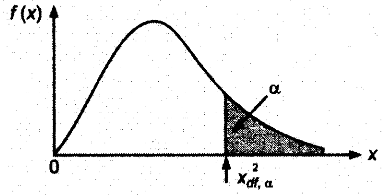
Value of z First decimal place x	Second Decimal									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2611	.2642	.2674	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4865	.4868	.4871	.4874	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Table A4: Critical Values of Student's *t*-Distribution



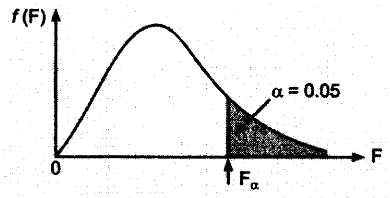
	Level of Significance for One-tailed Test				
d.f.	*0.10*	*0.05*	*0.025*	*0.01*	*0.005*
	Level of Significance for One-tailed Test				
d.f.	*0.20*	*0.10*	*0.05*	*0.02*	*0.01*
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

Table A5: Critical Values of Chi-Square (χ^2)



Degree of freedom	*0.100*	*0.050*	*0.025*	*0.010*	*0.005*	*0.001*
1	2.71	3.84	5.02	6.63	7.88	10.8
2	4.61	5.99	7.38	9.21	10.6	13.8
3	6.25	7.81	9.35	11.3	12.8	16.3
4	7.78	9.49	11.1	13.3	14.9	18.5
5	9.24	11.1	12.8	15.1	16.7	20.5
6	10.6	12.6	14.4	16.8	18.5	22.5
7	12.0	14.1	16.0	18.5	20.3	24.3
8	13.4	15.5	17.5	20.1	22.0	26.1
9	14.7	16.9	19.0	21.7	23.6	27.9
10	16.0	18.3	20.5	23.2	25.2	29.6
11	17.3	19.7	21.9	24.7	26.8	31.3
12	18.5	21.0	23.3	26.2	28.3	32.9
13	19.8	22.4	24.7	27.7	29.8	34.5
14	21.1	23.7	26.1	29.1	31.3	36.1
15	22.3	25.0	27.5	30.6	32.8	37.7
16	23.5	26.3	28.8	32.0	34.3	39.3
17	24.8	27.6	30.2	33.4	35.7	40.8
18	26.0	28.9	31.5	34.8	37.2	42.3
19	27.2	30.1	32.9	36.2	38.6	43.8
20	28.4	31.4	34.2	37.6	40.0	45.3
21	29.6	32.7	35.5	38.9	41.4	46.8
22	30.8	33.9	36.8	40.3	42.8	48.3
23	32.0	35.2	38.1	41.6	44.2	49.7
24	33.2	36.4	39.4	43.0	45.6	51.2
25	34.4	37.7	40.6	44.3	46.9	52.6
26	35.6	38.9	41.9	45.6	48.3	54.1
27	36.7	40.1	43.2	47.0	49.6	55.5
28	37.9	41.3	44.5	48.3	51.0	56.9
29	39.1	42.6	45.7	49.6	52.3	58.3
30	40.3	43.8	47.0	50.9	53.7	59.7
35	46.1	49.8	53.2	57.3	60.3	66.6
40	51.8	55.8	59.3	63.7	66.8	73.4
45	57.5	61.7	65.4	70.0	73.2	80.1
50	63.2	67.5	71.4	76.2	79.5	86.7

Table A6: Critical Values of the F-Distribution at a 5 Per cent Level of Significance



Degrees of freedom for the denominator	Degrees of freedom for the Numerator 1	2	3	4	5	6	7	8	9	10	12	15	20	30
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	250.09
2	18.513	19.000	19.164	19.247	19.296	19.330	193.53	19.371	19.385	19.396	19.413	19.420	19.446	19.462
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8868	8.8552	8.8123	8.7855	8.7446	8.7029	8.6602	8.6166
4	7.7086	6.9443	6.5914	6.3883	6.2560	6.1631	6.0942	6.0410	5.9988	5.9644	5.9117	5.8578	5.8025	5.7459
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8753	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.4957
6	5.9875	5.1433	4.7571	4.5337	4.3874	4.2839	4.2066	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8082
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5108	3.4445	3.3758
8	5.3177	4.4590	4.0662	3.8378	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2840	3.2184	3.1503	3.0794
9	5.1174	4.2565	3.8626	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0001	2.9365	2.8687
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.6996
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.5705
12	4.7272	3.8853	3.4903	3.2502	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.4663
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4579	2.3803
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6021	2.5342	2.4630	2.3879	2.3082
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4035	2.3275	2.2468
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.1938
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1477
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1071
19	4.3808	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.0712
20	4.3513	3.4928	3.0984	2.8661	2.7100	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0391
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3661	2.3210	2.2504	2.1757	2.0960	2.0102
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	1.9842
23	4.2793	3.4221	3.0280	2.7955	2.6500	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	1.9605
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9390
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9192
26	4.2252	3.3690	2.9751	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9010
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.8842
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900	2.1179	2.0411	1.9586	1.8687
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2782	2.2229	2.1768	2.1045	2.0275	1.9446	1.8543
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8409
31	4.0848	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772	2.0035	1.9245	1.8389	1.7444
32	4.0012	3.1504	2.7581	2.5252	2.3688	2.2540	2.1665	2.0970	2.0401	1.9926	1.9174	1.8364	1.7480	1.6491
33	3.9201	3.0718	2.6802	2.4472	2.2900	2.1750	2.0867	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.5543
34	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.4591

Table A7: Critical Values for Sign Test

n \ a	0.10 / 0.05	0.05 / 0.025	0.01 / 0.005	Two-sided tests / One-sided tests	n \ a	0.10 / 0.05	0.05 / 0.025	0.01 / 0.005	Two-sided tests / One-sided tests
5	0				23	7	6	4	
6	0	0			24	7	6	5	
7	0	0			25	7	7	5	
8	1	0	0		26	8	7	6	
9	1	1	0		27	8	7	6	
10	1	1	0		28	9	8	6	
11	2	1	0		29	9	8	7	
12	2	2	1		30	10	9	7	
13	3	2	1		31	10	9	7	
14	3	2	1		32	10	9	8	
15	3	3	2		33	11	10	8	
16	4	3	2		34	11	10	9	
17	4	4	2		35	12	11	9	
18	5	4	3		36	12	11	9	
19	5	4	3		37	13	12	10	
20	5	5	3		38	13	12	10	
21	6	5	4		39	13	12	11	
22	6	5	4		40	14	13	11	

Table A.8: Critical Values for the Wilcoxon Signed-Rank Test

n* \ a	0.10 / 0.05	0.05 / 0.025	0.01 / 0.005	Two-sided tests / One-sided tests
4				
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68

* If $n > 25$, W^+ (or W^-) is approximately normally distributed with mean $n(n + 1)/4$ and variance $n(n + 1)(2n + 1)/24$.

Table A.9: Critical Values for the Wilcoxon Rank-Sum Test (λ = 0.05)

n_2 \ n_1*	4	5	6	7	8	9	10	11	12	13	14	15
4	10											
5	11	17										
6	12	18	26									
7	13	20	27	36								
8	14	21	29	38	49							
9	15	22	31	40	51	63						
10	15	23	32	42	53	65	78					
11	16	24	34	44	55	68	81	96				
12	17	26	35	46	58	71	85	99	115			
13	18	27	37	48	60	73	88	103	119	137		
14	19	28	38	50	63	76	91	106	123	141	160	
15	20	29	40	52	65	79	94	110	127	145	164	185
16	21	31	42	54	67	82	97	114	131	150	169	
17	21	32	43	56	70	84	100	117	135	154		
18	22	33	45	58	72	87	103	121	139			
19	23	34	46	60	74	90	107	124				
20	24	35	48	62	77	93	110					
21	25	37	50	64	79	95						
22	26	38	51	66	82							
23	27	39	53	68								
24	28	40	55									
25	28	42										
26	29											
27												
28												

* For $n_1 > 8$, W_1 is approximately normally distributed with mean 1/2 $n_1 (n_1 + n_2 + 1)$ and varience $n_1 n_2 (n_1 + n_2 + 1)/12$.

Critical Values for the Wilcoxon Rank-Sum Test (λ = 0.01)

n_2 \ n_1	4	5	6	7	8	9	10	11	12	13	14	15
5		15										
6	10	16	23									
7	10	17	24	32								
8	11	17	25	34	43							
9	11	18	26	35	45	56						
10	12	19	27	37	47	58	71					
11	12	20	28	38	49	61	74	87				
12	13	21	30	40	51	63	76	90	106			
13	14	22	31	41	53	65	79	93	109	125		
14	14	22	32	43	54	67	81	96	112	129	147	
15	15	23	33	44	56	70	84	99	115	133	151	171
16	15	24	34	46	58	72	86	102	119	137	155	
17	16	25	36	47	60	74	89	105	122	140		
18	16	26	37	49	62	76	92	108	125			
19	17	27	38	50	64	78	94	111				
20	18	28	39	52	66	81	97					
21	18	29	40	53	68	83						
22	19	29	42	55	70							
23	19	30	43	57								
24	20	31	44									
25	20	32										
26	21											
27												
28												

Table A.10: Critical Values of R for the Runs Test (Lower Tail)

n_1 \ n_2	α = 0.026																		
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2	2	2	2	2	2	2	2	2
3					2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
4				2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
5			2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
6		2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
7		2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
8		2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
9		2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
10		2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
11		2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14

Critical Values of R for the Runs Test (Upper Tail)

n_1 \ n_2	α = 0.025																		
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2																			
3																			
4				9	9														
5			9	10	10	11	11												
6			9	10	11	12	12	13	13	13	13								
7				11	12	13	13	14	14	14	14	15	15	15					
8				11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9					13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10					13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11					13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12					13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13						15	16	17	18	19	19	20	20	21	21	22	22	23	23
14						15	16	17	18	19	20	20	21	22	22	23	23	23	24
15						15	16	18	18	19	20	21	22	22	23	23	24	24	25
16							17	18	19	20	21	21	22	23	23	24	25	25	25
17							17	18	19	20	21	22	23	23	24	25	25	26	26
18							17	18	19	20	21	22	23	24	25	25	26	26	27
19							17	18	20	21	22	23	23	24	25	26	26	27	27
20							17	18	20	21	22	23	24	25	25	26	27	27	28

Table A.11: p-Values for Mann-Whitney U Statistic Small Samples ($n_1 \leq n_2$)

$n_2 = 3$	U_0	n_1: 1	2	3
	0	.25	.10	.05
	1	.50	.20	.10
	2		.40	.20
	3		.60	.35
	4			.50

$n_2 = 4$	U_0	n_1: 1	2	3	4
	0	.2000	.0667	.0286	.0143
	1	.4000	.1333	.0571	.0286
	2	.6000	.2667	.1143	.0571
	3		.4000	.2000	.1000
	4		.6000	.3143	.1714
	5			.4286	.2429
	6			.5714	.3429
	7				.4429
	8				.5571

$n_2 = 5$	U_0	n_1: 1	2	3	4	5
	0	.1667	.0476	.0179	.0079	.0040
	1	.3333	.0952	.0357	.0159	.0079
	2	.5000	.1905	.0714	.0317	.0159
	3		.2857	.1250	.0556	.0278
	4		.4286	.1964	.0952	.0476
	5		.5714	.2857	.1429	.0754
	6			.3929	.2063	.1111
	7			.5000	.2778	.1548
	8				.3651	.2103
	9				.4524	.2738
	10				.5476	.3452
	11					.4206
	12					.5000

$n_2 = 6$	U_0	n_1: 1	2	3	4	5	6
	0	.1429	.0357	.0119	.0048	.0022	.0011
	1	.2857	.0714	.0238	.0095	.0043	.0022
	2	.4286	.1429	.0476	.0190	.0087	.0043
	3	.5714	.2143	.0833	.0333	.0152	.0076
	4		.3214	.1310	.0571	.0260	.0130
	5		.4286	.1905	.0857	.0411	.0206
	6		.5714	.2738	.1286	.0628	.0325
	7			.3571	.1762	.0887	.0465
	8			.4524	.2381	.1234	.0660
	9			.5476	.3048	.1645	.0898
	10				.3810	.2143	.1201
	11				.4571	.2684	.1548
	12				.5429	.3312	.1970
	13					.3961	.2424
	14					.4654	.2944
	15					.5346	.3496
	16						.4091
	17						.4686
	18						.5314

$N_1 = 7$ U_0	1	2	3	4	5	6	7
0	.1250	.0278	.0083	.0030	.0013	.0006	.0003
1	.2500	.0556	.0167	.0061	.0025	.0012	.0006
2	.3750	.1111	.0333	.0121	.0051	.0023	.0012
3	.5000	.1667	.0583	.0212	.0088	.0041	.0020
4		.2500	.0917	.0364	.0152	.0070	.0035
5		.3333	.1333	.0545	.0240	.0111	.0555
6		.4444	.1917	.0818	.0366	.0175	.0087
7		.5556	.2583	.1152	.0530	.0256	.0131
8			.3333	.1576	.0745	.0367	.0189
9			.4167	.2061	.1010	.0507	.0265
10			.5000	.2636	.1338	.0688	.0364
11				.3242	.1717	.0903	.0487
12				.3939	.2159	.1117	.0614
13				.4636	.2652	.1474	.0825
14				.5364	.3194	.1830	.1043
15					.3775	.2226	.1297
16					.4381	.2669	.1588
17					.5000	.3141	.1914
18						.3654	.2279
19						.4178	.2675
20						.4726	.3100
21						.5274	.3552
22							.4024
23							.4508
24							.5000

Table A. 12: Factors Useful in the Construction of Control Charts

Sample Size	Mean-Chart									Range-Chart			
	Factors for Control Limit			Factors for Central Line	Factors for Control Limit				Factors for Central Line	Factors for Control Limit			
n	A	A_1	A_2	c_2	B_1	B_2	B_3	B_4	d_2	D_1	D_2	D_3	D_4
2	2.121	3.760	1.881	0.6642	0	1.843	0	3.267	1.128	0	3.686	0	3.267
3	1.732	3.394	1.023	0.7236	0	1.858	0	2.566	1.693	0	4.358	0	2.575
4	1.500	2.880	0.729	0.7979	0	1.808	0	2.269	2.059	0	4.698	0	2.282
5	1.342	1.596	0.577	0.8407	0	1.756	0	2.089	2.326	0	4.918	0	2.115
6	1.225	1.410	0.483	0.8686	0.026	1.711	0.030	1.970	2.534	0	5.078	0	2.004
7	1.134	1.277	0.419	0.8882	0.105	1.672	0.118	1.888	2.704	2.205	5.203	0.076	1.924
8	1.061	1.175	0.073	0.9027	0.167	1.638	0.185	1.815	2.847	0.387	5.307	0.136	1.864
9	1.000	1.094	0.037	0.9139	0.219	1.609	0.239	1.761	2.970	0.546	5.394	0.184	1.816
10	0.949	1.028	0.308	0.9227	0.262	1.584	0.284	1.716	3.078	0.687	5.469	0.223	1.777
11	0.905	0.973	0.285	0.9300	0.299	1.561	0.321	1.679	3.173	0.812	5.534	0.256	1.744
12	0.866	0.925	0.256	0.9359	0.331	1.541	0.354	1.646	3.258	0.924	5.592	0.284	1.716
13	0.832	0.883	0.249	0.9410	0.359	1.523	0.382	1.618	3.336	1.026	5.646	0.308	1.692
14	0.802	0.848	0.235	0.9453	0.384	1.507	0.406	1.594	3.407	1.121	5.693	0.329	1.671
15	0.775	0.816	0.223	0.9490	0.406	1.492	0.428	1.572	3.472	1.207	5.737	0.348	1.652
16	0.750	0.788	0.212	0.9523	0.427	1.478	0.448	1.542	3.352	1.285	5.279	0.365	1.636
17	0.728	0.762	0.203	0.9551	0.445	1.465	0.466	1.534	3.588	1.359	5.817	0.379	1.621
18	0.707	0.738	0.816	0.9576	0.461	1.454	0.482	1.518	3.640	1.426	5.854	0.404	1.608
19	0.688	0.717	0.187	0.9599	0.477	1.443	0.497	1.503	3.689	1.490	5.888	0.404	1.596
20	0.671	0.697	0.180	0.9619	0.491	1.433	0.510	1.490	3.735	1.548	5.922	0.414	1.585
21	0.655	0.670	0.173	0.9638	0.504	1.424	0.523	1.447	3.778	1.606	5.950	0.425	1.575
22	0.640	0.662	0.167	0.9655	0.516	1.415	0.534	1.466	3.819	1.659	5.979	0.434	1.566
23	0.626	0.647	0.162	0.9670	0.527	1.407	0.545	1.455	3.858	1.710	6.006	0.443	1.557
24	0.612	0.632	0.157	0.9684	0.538	1.399	0.555	1.445	3.895	1.759	6.031	0.452	1.548
25	0.600	0.319	0.153	0.9696	0.548	1.392	0.565	1.435	3.931	1.804	6.058	0.459	1.541